LECTURES

Cristian Giardinà - Integrable models of heat conduction

This series of lectures aims to illustrate the connection between integrability and stochastic models of heat transport. This connection is rooted in the description of the generator of a stochastic process as the quantum Hamiltonian of a spin system. As we shall see, this connection is useful to better understand existing models, as well as to introduce new exactly solvable models.

In the first lecture, we introduce the notion of duality in the context of Markov processes and its relation to quantum spin chains, symmetry, and representation theory of Lie algebras.
In the second lecture, we discuss a few well-known heat conduction models, such as the Kipnis-Marchioro-Presutti (KMP) model, and the use of duality to characterize its non-equilibrium steady state.
In the third lecture, we present the recently introduced integrable version of the KMP model (so-called harmonic model) [1] and prove large deviations for the energy density [2]. If time permits, we will also discuss dynamical large deviations (ongoing work [3])

References:
[1] R. Frassek, C. Giardinà, J. Kurchan, *Non-compact quantum spin chains as integrable stochastic particle processes*, J. Stat. Phys. 180, 135–171 (2020); R. Frassek, C. Giardinà, *Exact solution of an integrable non-equilibrium particle system*, J. Math. Phys. 63, 103301 (2022).
[2] G. Carinci, C. Franceschini, R. Frassek, C. Giardinà, F. Redig, *Large Deviations and Additivity Principle for the Open Harmonic Process*, Comm. Math. Phys. 406 (2025).
[3] C. Giardinà, T. Sasamoto; *Large spin large deviations for interacting particle systems*; C. Giardinà, K. Mallick, T. Sasamoto, H. Suda, *Exact solution of discrete macroscopic fluctuation theory for an integrable spin system*, in preparation.

Lorenzo Piroli - Exactly solvable models of random quantum circuits

These lectures are meant for an audience familiar with statistical mechanics techniques, but will assume no prior knowledge of quantum circuits and quantum-information concepts. I will introduce recent applications of statistical mechanics and integrable techniques to the study of random quantum circuits. I will first give a brief and elementary introduction to the topic. I will in particular (1): briefly explain what are random quantum circuits and why they are currently of interest in the context of quantum information and computation theory, and (2): give an introduction to random quantum circuits as simple models for time-evolution in many-body systems. I will then focus on the prototypical computation of entanglement dynamics in random quantum circuits. I will explain how this problem can be mapped to an (integrable) statistical-mechanics partition function (a 2D classical Ising model). I will discuss generalizations and implications in the context of quantum computation theory and many-body theory. If time permits, I will finally describe how the model can be enriched by introducing external measurements and how this leads to novel phase transitions described by non-trivial critical exponents.

SEMINARS

Davide Lai - Clustering and the Five-Point Function

The hexagonalisation technique arose in the context of N=4 SYM theory to address the tessellation of the effective world-sheet describing three-point functions of single trace operators.
Despite its apparent technical difficulty, exploiting the knowledge of all its ingredients at finite 't Hooft coupling one can perform some non-trivial limits, like the strong-coupling one. In this case, a phenomenon called "clustering" allows the complete resummation of finite-size effects leading to the semi-classical string answer. I will introduce the general background and then discuss the generalization to some particular class of five-point functions, where the clustering phenomenon becomes more involved, involving cross-ratios and some analytic continuation of the ingredients to unusual kinematic regimes. In the end, I will speculate about some tentative directions that one could take.

Filiberto Ares - Entanglement asymmetry and the quantum Mpemba effect

The Mpemba effect is the counterintuitive and controversial phenomenon that hot water cools faster than cold one, or in more formal words, the more a system is out of equilibrium, the faster it relaxes. We introduce an analogous effect in extended quantum systems, in which a symmetry explicitly broken by the initial state is dynamically restored by the time evolution after a quantum quench. Unexpectedly, we find that the more the symmetry is initially broken, the faster is restored. To study this phenomenon, we borrow methods from the theory of entanglement in many-body systems and we introduce a quantity, dubbed entanglement asymmetry, which measures how much a symmetry is broken in a subsystem. In contrast to the classical case, we can establish the criteria for its occurrence in arbitrary integrable quantum systems and provide direct experimental evidence using a trapped-ion quantum simulator.

Niels Benedikter - Derivation of the Luttinger Model from the Interacting Fermi Gas

The Luttinger model is famous as a model of interacting fermionic quantum particles in one space dimension due to the fact that it can be exactly solved using the bosonization method discovered by Mattis and Lieb in 1965. This method however is very specific to the model with its linear dispersion relation and its particular form of interaction. I will discuss how the Luttinger model however arises in a mean-field limit of a general fermionic quantum many-body model which is not exactly solvable.

Francesco Ravanini - Hagedorn singularity in exact U_q(su(2)) S-matrix theories with arbitrary spins

Generalizing the quantum sine-Gordon and sausage models, we construct exact S-matrices for higher spin representations with quantum U_q(su(2)) symmetry, which satisfy unitarity, crossing-symmetry, and the Yang-Baxter equations with minimality assumption, i.e. without any unnecessary CDD factor. The deformation parameter q is related to a coupling constant. Based on these S-matrices, we derive the thermodynamic Bethe ansatz equations for q a root of unity in terms of a universal kernel where the nodes are connected by graphs of non-Dynkin type. We solve these equations numerically to find out Hagedorn-like singularities in the free energies at some critical scales and find a universality in the critical exponents, all near 0.5 for different values of the spin and the coupling constant.

Henrik Juergens - About the structure of the correlation functions of the sl(n+1) invariant vertex model

My talk will be about the structure of the correlation functions of the sl(n+1) invariant vertex model and it's corresponding spin chain. In the case n=1, i.e. the XXX and XXZ spin chain, there is a surprising result due to a series of papers of Jimbo et al. which ultimately allows the exact calculation of correlation functions both in the rational and trigonometric case through a 'hidden' fermionic structure. However, although this structure seems quite universal for calculating correlation functions for finite and infinite size, zero and finite temperature (and even form factors), there is to this day no clear understanding if or how this structure will generalise when considering higher rank symmetries. I will present an ansatz for generalising the initial construction of recursion relations for the correlation functions in the rational case (Jimbo et al. 2005) which has finally been published in April this year. As the topic is quite complicated, I will give a short overview about correlation functions, explain the 2005 construction and then focus on discussing the difficulties that come up when generalising. Moreover, I will conclude by interpreting the results in terms of so called snake modules - certain (prime) irreducible representations of the Yangian.

Francesco Buccheri - Orbital angular momentum control with Weyl semimetals

Twisted light beams [1] can carry large amounts of information over large distances, as well as selectively induce atomic and molecular transitions. Because of this, they hold significant promise for quantum computation and sensing technologies. In this talk, I will argue that Weyl semimetals, topological materials in which valence and conduction bands touch in a finite number of points [2], can be used to effectively manipulate twisted light. I will show that, because of the presence of an axionic term in the emergent electrodynamics [4,5], their surface plasmon nonreciprocity can be used to efficiently control the propagation of light in a cylindrical waveguide. As a consequence, a conical tip will focus light with given sign of the angular momentum only [5].

[1] L. Allen, M. W. Beijersbergen, R. J. C. Spreeuw, and J. P. Woerdman, "Orbital angular momentum of light and the transformation of Laguerre-Gaussian laser modes" , Phys. Rev. A 45, 8185
[2] N. P. Armitage, E. J. Mele, and A. Vishwanath, "Weyl and Dirac semimetals in three-dimensional solids", Rev. Mod. Phys. 90, 015001 (2018);
[3] A. A. Zyuzin and A. A. Burkov, "Topological response in Weyl semimetals and the chiral anomaly", Phys. Rev. B 86, 115133 (2012);
[4] M. M. Vazifeh and M. Franz, Electromagnetic response of Weyl semimetals, Phys. Rev. Lett.

111, 027201 (2013);
[5] M. Peluso, A. De Martino, R. Egger, and F. Buccheri, "Nonreciprocal Weyl semimetal waveguide", arXiv:2410.01503 (2024) & "Selective plasmon nanofocusing with Weyl semimetals", in preparation.

Vanja Marić - Disorder-Order Interface Propagating over the Ferromagnetic Ground State

We consider translationally invariant quantum spin-1/2 chains with local interactions and a discrete symmetry that is spontaneously broken at zero temperature. We envision experimenters switching off the couplings between two parts of the system and preparing them in independent equilibrium states. One side of the chain is prepared in a symmetry-breaking ground state and the other side in a disordered phase, for example equilibrium at higher temperature. When the couplings are switched back on, time evolution ensues. We argue that in integrable systems the front separating the ordered region recedes at the maximal velocity of quasiparticle excitations over the ground state. We infer that, generically, the order parameters should vary on a subdiffusive scale of order $t^{1/3}$, where t is time, and their fluctuations should exhibit the same scaling. Thus, the interfacial region exhibits full range correlations, indicating that it cannot be decomposed into nearly uncorrelated subsystems. Using the transverse-field Ising chain as a case study, we demonstrate that all order parameters follow the same universal scaling functions. Additionally, we present data on Rényi entanglement asymmetries and a prediction valid also in the von Neumann limit. Through an analysis of the skew information, we uncover that the breakdown of cluster decomposition has a quantum contribution: each subsystem within the interfacial region, with extent comparable to the region, exists in a macroscopic quantum state.

References: arXiv:2410.10645, Phys. Rev. Lett. (to appear),

arXiv:2411.04089, Phys. Rev. B 111, 205118

Marco Meineri - No-go theorems for higher-spin charges in AdS_2

Higher-spin conserved currents and charges feature prominently in integrable 2d QFTs in flat space. Motivated by the question of integrable field theories in Anti de-Sitter space, we consider the consequences of higher-spin currents for QFTs in AdS2, and find that their effect is much more constraining than in flat space. Specifically, it is impossible to preserve: (a) any higher-spin charges when deforming a massive free field by interactions, or (b) any spin-4 charges when deforming a CFT by a Virasoro primary. Along the way, we explain how higher-spin charges lead to integer spacing in the spectrum of primaries and to constraints on correlation functions. We also comment on consequences for critical long-range statistical models.

Saverio Rota - A Bethe ansatz approach to the Thirring Quantum Cellular Automaton

We investigate the integrability of the 1+1 dimensional massless Thirring Quantum Cellular Automaton, which models the discrete-time evolution of fermionic modes on a lattice through local, number-preserving interactions. These interactions act as a discrete-time analogue of those found in integrable Hamiltonian systems such as the Thirring and Hubbard models. Motivated by this correspondence, we apply the coordinate Bethe ansatz—a well-established method in the study of integrable systems—to analyze the spectrum of the unitary operator governing the automaton's update rule.

Our goal is to construct translationally invariant eigenstates, assuming they can be expressed as plane waves with momenta permuted among particles—an assumption we verify a posteriori by testing the completeness of the resulting solutions. We have obtained explicit solutions for the two- and three-particle sectors and are currently working toward a generalization for arbitrary particle numbers. Ultimately, our aim is to recover the Yang-Baxter equation, which would allow for a recursive construction of the solutions in terms of a representation of the symmetric group.

Our preliminary findings indicate that the periodicity of the quasi-energy—a consequence of the model's discrete-time nature—necessitates a slight modification of the standard Bethe ansatz used in the Hamiltonian setting. In particular, the wavefunction must include additional components in its plane-wave expansion, corresponding to quasi-energies that differ by integer multiples of 2pi.

Andrea Appel - Quantum symmetric pairs and boundary integrability

Representation theory of quantum algebras provides a natural algebraic framework for integrability. While the Yang-Baxter equation is deeply tied to Drinfeld-Jimbo quantum groups, solutions of the reflection equation arise naturally in the theory of quantum symmetric pairs. In this talk, I will survey recent results from joint works with B. Vlaar, emphasizing the role of quantum symmetric pairs in constructing and classifying integrable boundary conditions.

Lenart Zadnik - Integrability and transport in magnetization-conserving nearest-neighbour qubit circuits

Integrability is considered an exceptional property, believed to hold only for systems with fine-tuned parameters. I will show that this is not the case in nearest-neighbour qubit circuits with a U(1) symmetry. There, integrability is generic: all circuits in which a magnetization-conserving unitary gate is applied to each pair of neighbouring qubits exactly once per period are integrable, even if the gate is randomly selected. An experimentally relevant feature of such circuits is a phase transition that can be crossed by varying any one of the parameters in the generic U(1) gate. It separates two phases that differ in symmetries, conservation laws, transport properties, and zero edge modes. I will show how various integrable circuit geometries can be constructed, describe their spectrum and its quasiparticle content, and discuss how circuit geometry affects the magnetization transport.

Davide Fioravanti - The importance of being Exact: gauge, integrability, black holes physics and their correspondence.

Schroedinger-like equations (Sle) in one dimension have been playing a mayor rôle in modern theoretical physics. In fact, they enter many concrete and even experimental problems in higher dimensions, usually as a reduction, and are suitable for powerful semi-classical analysis since the very beginning of Quantum Mechanics when this method has started shading light on the deep meaning of quantisation. In this respect, one of the most important appearance of Sle equations (of Heun-type, with regular and irregular singularities) occurs, in recent times, in the theory of General Relativity (GR) perturbation by scalars or tensor excitations: with particular relevance for Black Holes (BHs) and their scattering, as they produce ultimately Gravitational Waves. In specific, what matters for theoretical understanding and experimental data is the spectrum of the so-called Quasi Normal Modes (QNMs), the characteristic frequencies of a gravitational object, like a BH. Actually, important is also the computation of the associated wave-functions and Floquet functions, being the latter associated to the Love numbers. A very new method for this investigation has become, more and more, the use of the Nekrasov-Shatashvily partition functions of susy gauge theories. Moreover, it has been shown to be captured by a deep integrability structure encompassing the (direct and inverse) monodromy problem of the Sle. The originality and efficiency of these new powerful idea and method will be described in computing exactly QNMs — via new thermodynamic Bethe Ansatz equations —, the related eigen-functions and the Floquet perturbations. As a byproduct these Sle are produced as limit of the isospectral Painlevé flow to the pole.